RAN-1183

RAN-1183

T.Y.B. Sc. Sem -VI Examination

March / April - 2019

Mathematics Paper : MTH - 601

Ring Theory

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
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Name of the Examination:
☛ T.Y.B. Sc. Sem -VI

Name of the Subject :
☛ Mathematics Paper : MTH - 601

Subject Code No.: 1 1 8 3

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

1. Answer the following as directed : (Any FIVE) **(10)**

(1) $R = \{\bar{0}, \bar{2}, \bar{4}, \bar{6}, \bar{8}\}$ is a commutative ring under the binary operations $+_{10}$ (addition modulo 10) and x_{10} (multiplication modulo 10).
Justify : Every non-zero element in this R has an inverse for x_{10} .

(2) In a ring R; prove that $a.(-b) = -(a.b)$; for all $a, b \in$ R.

(3) Mention all the ideals of the ring J_{13} ; of integers modulo 13.

(4) If U is an ideal of a ring R with a unit element 1 and $1 \in U$, then prove that $U = R$.

(5) Prove that $\bar{3} \mid \bar{5}$ and $\bar{5} \mid \bar{3}$ in the commutative ring J_8 ; of integers modulo 8.

(6) Let R be a Euclidean ring and $a \neq 0$, $b \neq 0$ in R. If b is unit in R, then prove that $d(a) = d(a.b)$.

(7) Justify : $\bar{4}$ and $\bar{8}$ are relatively prime elements in the Euclidean ring J_{11}; of integers modulo 11.

(8) Define a prime element in a Euclidean ring . Which are the prime elements in the Euclidean ring J_7 ; of integers modulo 7 ?

2. **Attempt any TWO :**

(1) Prove that every finite integral domain is a field.

(2) Prove that the commutative ring D is an integral domain if and only if $a, b, c \in$ D with $a \neq 0$; the relation $a. b = a. c$ implies that $b = c$ holds in D.

(3) Define a Boolean ring. Prove that every Boolean ring is commutative.

3. **Attempt any TWO :**

(1) Define the Kernel of a homomorphism. Let $\phi : R \rightarrow R'$ be a homomorphism of a ring R into a ring R'. Then prove that: $\phi(0) = 0'$ and $\phi(-a) = -\phi(a)$; for every a in R.

(2) Prove that a homomorphism $\phi : R \rightarrow R'$ of a ring R into a ring R' is an isomorphism if and only if $I(\phi) = (0)$; where $I(0)$ is the Kernel of a homomorphism ϕ .

(3) If R is a commutative ring with a unit element 1 and its only ideals are (0) and R itself, then prove that R is a field.

4. **Attempt any TWO :** (10)

(1) Define a Euclidean ring. Prove that every field is a Euclidean ring.

(2) Prove that the relation of "*associates*" in a commutative ring R with a unit element is an equivalence relation on R.

(3) Define a greatest common divisor of two elements in *a* commutative ring. Prove that any two greatest common divisors of elements a, b in a Euclidean ring R are associates.

5. **Attempt any TWO :** (10)

(1) Define relatively prime elements in a Euclidean ring. If a and b are relatively prime elements in a Euclidean R and $a \mid bc$, then prove that $a|c$.

(2) Let R be a Euclidean ring. If $A = (a_0)$ is a maximal ideal of R, then prove that a_0 is a prime element in R.

(3) Define unit in a commutative ring with a unit element. Let R be a Euclidean ring and $a \neq 0$, $b \neq 0$ in R. If b is not unit in R, then prove that $d(a) < d(a. b)$.

RAN-1184

RAN-1184

Third Year B. Sc. (Mathematics) (Sem. VI) Examination

March / April - 2019

MTH-602-Linear Algebra-II

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

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Name of the Examination:
☛ **Third Year B. Sc. (Mathematics) (Sem. VI)**

Name of the Subject :
☛ **MTH-602-Linear Algebra-II**

Subject Code No.: 1 1 8 4

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the questions.

(4) Follow usual notations.

Q. 1. **Answer the following questions (Any Five).** **(10)**

(1) Define : Identity map in a vector space .Prove that it is linear .

(2) Is a transformation $T: V_2 \to V_3$ defined by $T(1,1) = (1,0,0)$, $T(2,1) = (0,0,1)$ and $T(0,1) = (0,1,0)$ linear? Justify your answer.

(3) Obtain the general rule for a linear map

$$T : V_2 \to V_4;\ T(1,1) = (1,1,0,0),\ T(1,-1) = (0,0,0,0).$$

(4) Is the following linear transformation one-one ? Justify your answer.

$$T : V_2 \to V_4;\ T(1,1) = (1,0,0,0),\ T(1,2) = (2,0,0,0).$$

(5) Prove that a linear transformation $T : V_2 \rightarrow V_2$ define by $T(x,y) = (x, -y)$ is non singular.

(6) Prove that A linear transformation $T : U_p \rightarrow V_p$ with $r(T) = p$ then $n(T) = 0$.

(7) Find the range and rank of the matrix $\begin{bmatrix} 2 & 1 \\ -2 & -1 \end{bmatrix}$.

(8) State two difference between Orthogonal and Orthonormal set in an Inner product space V.

Q. 2. Answer the following (Any two). **(10)**

(1) State and prove necessary and sufficient condition for a linear transformation $T:U \rightarrow V$ be 1-1.

(2) Define Null space. Let $T:U \rightarrow V$ be a linear map. If $u_1 u_2, u_3, \ldots \ldots, u_n$ are linearly independent vectors of U with $N(T) = \{\theta_U\}$ then prove that $T(u_1), T(u_2), T(u_3) \ldots \ldots T(u_n)$ are L.I.

(3) Obtain the general rule of the linear transformation $T : V_3 \rightarrow V_3$ defined by $T(0,1,2) = (3,1,2)$, $T(1,1,1) = (2,2,2)$, $T(0,1,3) = (3,1,2)$.

Q. 3. Answer the following (Any two). **(10)**

(1) Verify Rank -Nullity theorem for a linear transformation $T: V_3 \rightarrow V_3$ defined by

$T(e_1) = e_1 - e_2$, $T(e_2) = 2e_2 + e_3$ and $T(e_3) = e_1 + e_2 + e_3$.

(2) Let $T:U \rightarrow V$ and $S:V \rightarrow W$ be two linear maps. Then prove that

(a) If ST is one-one, then T is one-one.

(b) If ST is non singular, then T is one-one and S on-to.

(3) Find the inverse of the linear transformation $T: V_3 \rightarrow V_3$ defined by $T(e_1) = e_1 + e_2$, $T(e_2) = e_2 - e_2 + e_3$ and $T(e_3) = 3e_1 + 4e_3$.

Q. 4. Answer the following (Any two). **(10)**

(1) Find the matrix $(T; B_1 B_2)$ associated with a linear transformation $T : V_3 \to V_2$ defined by $T(X, Y, Z) = (2X + Y, 2Y - Z)$ relative to basis $B_1 = \{(1,1,0), (1,0,1), (0,1,1)\}$ and $B_2 = \{(1,1), (1,-1)\}$.

(2) Verify Rank-Nullity theorem for the matrix $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix}$

(3) Find the Linear transformation T associated with a matrix $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ relative to basis $B_1 = \{(1,-1,1), (1,2,0), (0,-1,0)\}$ and $B_2 = \{(1,0), (2,-1)\}$.

Q. 5. Answer the following (Any two). **(10)**

(1) In an Inner product space V, prove that

(a) $\|u + v\| \leq \|u\| + \|v\|$, $\forall\, u, v \in V$.

(b) $u \cdot (\alpha v) = \bar{\alpha}\,(u \cdot v)$, $\forall\, u, v \in V$ and α a scalar.

(2) (a) Prove that any orthogonal set of non zero vectors in an inner product space V is L.I.

(b) Explain : Euclidean space and Unitary space in inner product space.

(3) Orthonormalized the L.I set $\{(0,0,1),(1,1,0),(1,5,2)\}$ by Gram Schmidt's process.

RAN-1185

RAN-1185

T.Y.B.Sc.(Mathematics) (Sem-VI) Examination

March / April - 2019

Paper-603 Real Analysis III

(Old or New to be mentioned where necessary)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

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(2) Figures to the right indicate marks of the question.

(3) Follow usual notations and conventions.

Q.1 **Answer any FIVE from the following.** **[10]**

1. Define convergence and conditional convergence of a series of real numbers.
2. Define (i) Harmonic series (ii) Alternating series.
3. Give statement of the "ROOT TEST" for the absolute convergence of the series of real numbers.
4. Prove that the series $\sum_{n=2}^{\infty} \frac{1}{n^2}$ is convergent.
5. Prove that a singleton set is of measure zero.
6. Define Upper Riemann Integral and Lower Riemann Integral for a bounded function f on $[a, b]$.

7. If $f(x)=\int_0^x \sqrt{t+t^6}\,dt\ (x>0)$ then find $f'(2)$.

8. If f is continuous on $[a, b]$, then prove that there exists $c \in (a, b)$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$.

Q.2 **Attempt any Two.** **[10]**

(a) If $\sum_{n=2}^{\infty} a_n$ is a convergent series then prove that $\lim_{n\to\infty} a_n = 0$. Is converse true ? Justify your answer.

(b) Check the convergence of following series.

(i) $1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+\ldots$ (ii) $\sum_{n=1}^{\infty}\frac{(n+1)}{(n+2)}$

(b) Prove that the series $(1-2)-(1-2^{\frac{1}{2}})+(1+2^{\frac{1}{3}})-(1-2^{\frac{1}{4}})+\ldots$ converges.

Q.3 **Attempt any Two.** **[10]**

(a) If $\{a_n\}_{n=1}^{\infty}$ is a non-increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} 2^n a_{2n}$ di-verges then prove that $\sum_{n=1}^{\infty} a_n$ diverges.

(b) Using appropriate test of convergence check the convergence for the series $\sum_{n=1}^{\infty}\frac{3}{4+2n}$

(c) For what, values of x does the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^x}$ converge ?

Q.4 **Attempt any Two.** **[10]**

(a) If each of the subsets $E_1, E_2, E_3, \ldots$ of R^1 is of measure zero, then prove that $\bigcup_{n=1}^{\infty} E_n$ is also of measure zero.

(b) Prove that the characteristic function of the set of rational numbers on $[a, b]; a<b$ is not Riemann Integrable.

(c) Let $f(x)=x\ (0\le x\le 1)$ and $\sigma_n=\left\{0, \frac{1}{n}, \frac{2}{n}, \frac{3}{n}, \ldots, \frac{n}{n}\right\}$ be any subdivision of $[0,1]$ then compute $\lim_{n\to\infty} U[f;\sigma_n]$

Q.5 Attempt any Two. **[10]**

(a) If f is a continuous function on the closed bounded interval $[a, b]$, and if $\Phi'(x) = f(x)\,(a \le x \le b)$, then prove that $\int_a^b f(x)\,dx = \Phi(b) - \Phi(a)$.

(b) If f is a continuous on $[a, b]$ and if $F(x) = \int_a^x f(t)\,dt\;(a \le x \le b)$, then prove that F is also continuous on $[a, b]$.

(c) If $f \in R[a, b]$, then prove that $|f| \in R[a,b]$ and $\left|\int_a^b f\right| \le \int_a^b |f|$.

RAN-1186

B. Sc. Sem -VI Examination

March / April - 2019

Mathematics Paper : MTH - 604

Real Analysis - IV

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

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Name of the Examination:
☛ B. Sc. Sem -VI

Name of the Subject :
☛ Mathematics Paper : MTH - 604

Subject Code No.: 1 1 8 6

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

1. **Answer the following as directed : (Any FIVE)** **(10)**

(1) Prove that every finite set in any metric space is closed.

(2) Justify: (2019,2020) is a closed subset of the metric space $\langle(2019,2020), |.|\rangle$.

(3) Justify: $[0,1] \cup [2,3]$ is a connected set of the metric space R^1.

(4) Prove that $(0, \infty)$ is a bounded subset of the metric space R_d and its diameter is 1.

(5) Justify : R_d is not the complete metric space.

(6) State Picard's Fixed - Point Theorem.

(7) (i) Give an example of a compact subset of R^1 which is not connected;
(ii) Give an example of a subset of R_d which is compact as well as connected.

(8) State Finite - Intersection property and give its illustration in the me[tric] space R^1.

2. **Attempt any TWO :**

(1) Let E be the subset of a metric space $\langle M, \rho \rangle$. Prove that $\overline{E}$; the closu[re] of E; is closed.

(2) Define a closed set in a metric space. Prove that a finite intersection o[f] closed sets in any metric space is closed.

(3) If A and B are closed subsets of R^1, then prove that $A \times B$ is a closed subset of R^2.

3. **Attempt any TWO :**

(1) If the metric space M is connected, then prove that every continuous characteristic function on M is constant.

(2) If A is a connected subset of a metric space $\langle M, \rho \rangle$, then prove that $\overline{A}$ is also connected.

(3) Define a totally bounded set. If A is a totally bounded subset of the metri[c] space R_d, then prove that A contains only a finite number of points.

4. **Attempt any TWO :**

(1) Prove that a closed subset of a complete metric space is complete.

(2) Prove that R^2 is a complete metric space ; with respect to the metric τ for R^2 defined as : $\tau(P,Q) = max\{|x_1 - x_2|, |y_1 - y_2|\}$; where $P = \langle x_1, y_1 \rangle$ & $Q = \langle x_2, y_2 \rangle$ in R^2.

(3) Define a contraction mapping. Prove that a mapping

$$T : \left\langle \left(0, \tfrac{1}{3}\right], |\cdot| \right\rangle \to \left\langle \left(0, \tfrac{1}{3}\right], |\cdot| \right\rangle$$

defined by $Tx = x^2$; for every $x \in (0, \frac{1}{3}]$; is contraction, but it does not have a fixed point.

5. **Attempt any TWO :**

(1) Define a compact metric space. Prove that a closed subset of a compact metric space is compact.

(2) Prove that: (i) Every finite set in any metric space is compact.

(ii) A connected subset of the metric space R_d is compact.

(3) If the metric space M has the Heine -Borel Property, then prove that M is compact.

RAN-1187

B.Sc. Sem-VI Examination

March / April - 2019

MTH-605-Mathematics

(Discrete Mathematics)

(Old or New to be mentioned where necessary)

[Total Marks: 50

સૂચના : / Instructions

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Name of the Examination:

☛ **B.Sc. Sem-VI**

Name of the Subject :

☛ **MTH-605-Mathematics**

Subject Code No.: 1 1 8 7

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Follow usual notations.
(3) Figures to the right indicate marks of the question.

Que:1 **Answer any FIVE as directed.** **[10]**

(1) State and prove absorption law with respect to meet and join operations.

(2) Define : Lattice Homomorphism.

(3) State modular inequality in a lattice.

(4) Write the Boolean expression $(x_1 * x_2)$ in the sum of the products canonical form in the variables x_1 x_2 and x_3.

(5) Define sub lattice with one illustration.

(6) Show that 1 is the only complement of 0.

(7) In Boolean algebra, prove that $a \oplus (a' * b) = a \oplus b$

(8) In a Boolean Algebra, prove that
$a \leq b \Rightarrow a + b \,.\, c = b \,.\, (a + c)$.

Que:2 **Answer the following (any TWO).** [1(

(1) Define a partially ordered relation. Prove that $\langle p(A), \subseteq \rangle$ is a partially ordered set. Where $\rho(A)$ is a power set of A and define the relation $\subseteq$ (inclusion).

(2) Let $X = \{1, 2, 3, 4, 6, 8, 12, 24, 48\}$ and the relation " $\leq$ " be the divides. Draw the Hasse diagram of $< X, \leq >$. Is it a sub lattice of $< I_+, D >$? Justify

(3) Let R denote a relation on the set of ordered pairs of positive integers such that $\langle x\ y \rangle\ R\ \langle u, v \rangle$ if and only if $xv = yu$. Show that R is an equivalence relation.

Que:3 **Answer the following (any TWO).** [10]

(1) Let $(L, \leq)$ be a lattice. For any $a, b, c \in L$, prove that

(a) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$

(b) $a * (b \oplus c) \geq (a * b) \oplus (a * c)$

(2) Let $< B, *, \oplus, ', 0, 1 >$ is a Boolean algebra. Let S be a non empty subset of B. If S preserving the operations $\oplus$ and $'$ then prove that $< S, *, \oplus, ', 0, 1 >$ is a sub-boolean algebra.

(3) Let $(L, \leq)$ be a lattice. For any $a, b, c \in L$, prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$

Que:4 **Answer the following (any TWO).** [10]

(1) Obtain the sum of products canonical form of the Boolean expression $x_1 \oplus (x_2 * x_3')$.

(2) Simplify the following Boolean expressions:

(a) $(a * b)' \oplus (a \oplus b)'$

(b) $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$

(3) Let $\langle B, *, \oplus, ', 0, 1 \rangle$ be a Boolean Algebra. Define the operations ' + ' and ' $\cdot$ ' on the elements of B by $a + b = (a * b') \oplus (a' * b)$ and $a \cdot b = a * b$; then prove that

(a) $a + a = 0$

(b) $(a + b) \oplus a \cdot b = a * b$

(c) $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

Que:5 **Answer the following (any TWO).** **[10]**

(1) Use Karnaugh map representation to find the minimal sum of products of the function $f(a, b, c, d) = \Sigma\,(5,7,10,13,15)$

(2) Use Quine McCluskey algorithm to find the minimal sum of products form of $f(a, b, c, d) = \Sigma(10,12,13,14,15)$.

(3) Find the minimal sum of products of the function $f(a, b, c, d) = \Sigma(0,2,6,7,8,9,13,15)$ by using Karnaugh map representation.

RAN - 1188

RAN-1188

B.Sc (Sem.-V) Examination

March / April - 2019

Mathematics : Paper -606

(Number Theory)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

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Name of the Examination:
☛ **B.Sc (Sem.-V)**
Name of the Subject :
☛ **Mathematics : Paper -606 (Number Theory)**
Subject Code No.: 1 1 8 8

Seat No.:

Student's Signature

(2) Digits to the right indicates marks of the question.

(3) Follow the usual notations.

Q-1 **Answer any five questions :** **(10)**

(1) Show that 561 is a Pseudo prime .

(2) Arrange the integers 2, 3, 4, ------ , 21 in pairs a and b such that $ab \equiv 1 \ (mod\ 23\ )$.

(3) Show that $a^{21} \equiv a$ (mod 15) for any integer a.

(4) Prove that $\sum_{k=1}^{n} \mu(k!) = 1$ where $n \geq 3$.

(5) Show that $\tau(m + 1) = \tau(m + 2)$ for $m = 3655$.

(6) Solve : $36x \equiv 8(mod\ 102)$.

(7) Show that $[x] + [-x] = 0$ or -1 according as x is an integer or not

(8) Prove that $\phi(n) + 2 = \phi(n + 2)$ for $n = 4p$ where p and $2p + 1$ both are odd primes.

Q-2 **Answer any two questions :**

(1) State and prove Chinese Remainder Theorem .

(2) Solve the linear congruence $34x \equiv 60 (mod\ 98)$.

(3) Obtain three consecutive integers, the first of which is divisible by fourth Power, the second by a cube and third by a square.

Q-3 **Answer any two questions :**

(1) If n is an odd pseudo prime then prove that $M_n = 2^n - 1$ is larger one.

(2) If p and q are distinct primes with $a^p \equiv a\ (mod\ q)$ and $a^q \equiv a\ (mod\ p)$ then prove that $a^{pq} \equiv a$ (mod pq).

(3) If p is a prime then for any integer a prove that

(i) $p | a^p + (p-1)!a$ (ii) $p | (p-1)!a^p + a$

Q-4 **Answer any two questions :**

(1) Prove that τ and σ functions are multiplicative .

(2) For any positive integer n; show that $\mu(n).\mu(n+1).\mu(n+2).\mu(n+3) =$ (

(3) Find the highest power of 65 which divides 1000!

Q-5 **Answer any two questions :**

(1) If m and n are relatively prime positive integer then prove that $m^{\phi(n)} + n^{\phi(m)} \equiv 1 (\text{mod } mn)$.

(2) Prove that $\varphi(n) = \frac{n}{2}\ iff\ n = 2^k$ for some $k \geq 1$.

(3) Prove that $\varphi(2n) = \varphi(n)\ or\ \varphi(2n) = 2\varphi(n)$ according as n is an odd Or even integer.

RAN-1189

RAN-1189

B.Sc. Sem-VI Examination

March / April - 2019

Mathematics-MTH-6001 (EG)

(Operations Research-II)

સૂચના : / Instructions

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Name of the Examination:

☛ **B.Sc. Sem-VI**

Name of the Subject :

☛ **Mathematics - MTH - 6001 (EG)**

Subject Code No.: 1 1 8 9

Seat No.:

Student's Signature

Instruction:

(1) All questions are compulsory.
(2) Figures to the right indicate marks of the question.
(3) Follow usual notations.
(4) Use of non-programmable calculator is allowed.
(5) Total marks **50**.

Que:1 (a) Answer any TWO as directed. **[06]**

(1) Write two applications of the assignment problem.

(2) Solve the following Assignment problem:

	I	*II*	*III*
A_1	10	8	6
A_2	6	7	9
A_3	9	12	10

(3) Write the general mathematical form of Transportation problem.

Que:1 (b) Attempt any ONE.

(1) Consider the game with following payoff table. Determine the value of the game.

Player A	Player B: B1	B2
A_1	7	–2
A_2	5	4

(2) Consider the game with following payoff table. Determine the value of the game.

Player A	Player B: B1	B2
A_1	–3	4
A_2	2	–1

Que:2 Attempt any TWO. [20]

(1) Find the assignment of workers to machines that will minimize the total time taken.

Manufacturers	*Machines*: M_1	M_2	M_3	M_4	M_5
A_1	25	28	29	28	31
B_2	31	29	30	31	29
C_3	27	26	28	27	26

(2) Solve the Assignment Problem:

Employee	*Jobs*: J_1	J_2	J_3	J_4	J_5
E1	5	5.1	4.2	5.7	4.9
E2	5.1	1.5	5.8	6	4.3
E3	6.5	5.5	4.6	6.4	6

[04

(3) Solve the Assignment Problem:

		Salesmen				
		A	*B*	*C*	*D*	*D*
Counters	*I*	25	30	38	50	15
	II	28	26	35	50	20
	III	30	35	40	55	18
	IV	15	25	30	48	12
	V	30	27	32	48	16

(4) Use graphical method to solve the following game and find the value of the game.

		Player B			
		B_1	B_2	B_3	B_4
Player A	A_1	1	4	6	8
	A_2	8	3	4	2

[20]

Que:3 Attempt any TWO. **[20]**

(1) Find an initial basic feasible solution for the following Transportation problem using
(i) North west corner method
(ii) Least cost method.

		Destinations				
		D_1	D_2	D_3	D_4	Supply
Sources	S_1	21	6	15	3	**110**
	S_2	17	18	4	23	**130**
	S_3	32	27	18	14	**190**
Demand		**60**	**100**	**120**	**150**	

(2) Determine an optimal solution for the following transportation problem using MODI method:

		Destinations				
		D_1	D_2	D_3	D_4	Supply
Sources	S_1	3	6	8	5	**20**
	S_2	6	1	2	5	**28**
	S_3	7	8	3	9	**17**
Demand		**15**	**19**	**13**	**18**	

(3) Solve the following transportation problem:

		Destinations					
		D_1	D_2	D_3	D_4	D_5	**Supply**
Sources	S_1	5	3	4	6	4	**4**
	S_2	4	3	10	5	6	**2**
	S_3	4	6	9	4	3	**4**
Demand		**2**	**1**	**2**	**3**	**2**	

(4) Use graphical method to solve the following game and find the value of the game.

		Player B	
		B_1	B_2
Player A	A_1	2	5
	A_2	4	6
	A_3	3	3
	A_4	8	7
	A_5	4	8
	A_6	5	4

RAN-1042

RAN-1042

T.Y.B.Sc. (Mathematics) (Sem. V) Examination

March / April - 2019

Paper - 503 Real Analysis I

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ T.Y.B.Sc. (Mathematics) (Sem. V)

Name of the Subject :

☛ Paper - 503 Real Analysis I

Subject Code No.: 1 0 4 2

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Digits to the right of each question indicate its marks.
(3) Follow usual symbols.

Q. 1 Answer any FIVE from the following. **[10]**

1) Define the upper bound of a set and find g.l.b. for the set $\{\pi+1, \pi+2, \pi+3, \pi+4, ...\}$

2) Prove that if $\{|S_n|\}_{n=1}^{\infty}$ converges to 0 then $\{S_n\}_{n=1}^{\infty}$ converges to 0.

3) If $S=\{S_n\}_{n=1}^{\infty}=\{2n-1\}_{n=1}^{\infty}$ and $N=\{n_i\}_{i=1}^{\infty}=\{i^2\}_{i=1}^{\infty}$ then find S_8 and S_{n_4}.

4) If $\{S_n\}_{n=1}^{\infty}$ is a sequence of real numbers, if $S_n \le M\ (n \in I)$ and if $\lim_{n\to\infty} s_n = L$ then prove that $L \le M$..

5) Classify the sequence $\left\{n\sin\frac{\pi}{n}\right\}_{n=1}^{\infty}$ into

(A) convergent (B) divergent to ∞

(C) divergent to $-\infty$ (D) oscillating.

6) Define limit superior for a sequence of real numbers and find it for 12,–3,1,2,–3,12,–3,1,2,–3,........

7) Define a Cauchy sequence of real numbers with an illustration.

8) Give an example of a sequence $\{S_n\}_{n=1}^{\infty}$ which is not bounded but for which $\lim_{n\to\infty} \frac{S_n}{n} = 0$.

Q. 2 Answer any TWO from the following. **[1**

1) Show that the set of all ordered pairs of integers is countable.

2) Prove that the set $A_n = \left\{\frac{m}{n} : m \in I\right\}$ is countable for each $n \in N$, where I is the set of integers and use it to show that the set of all rational numbers is countable.

3) Define finite set with an illustration and prove that if B is an infinite subset of a countable set A, then B is also countable.

Q. 3 Answer any TWO from the following. **[1**

1) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent to L, then prove that my subsequence of $\{S_n\}_{n=1}^{\infty}$ is also convergent to L.

2) Suppose $\{S_n\}_{n=1}^{\infty}$ converges to L then prove that $\{(-1)^n S_n\}_{n=1}^{\infty}$ converges to 0 if L = 0 and $\{(-1)^n S_n\}_{n=1}^{\infty}$ oscillates if $L \neq 0$.

3) If the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ converges to L, then prove that $\{S_n\}_{n=1}^{\infty}$ cannot also coverge to a limit distinct from L.

Q. 4 Answer any TWO from the following. **[1**

1) Define a bounded sequence of real numbers and if the sequence of real numbers $\{S_n\}_{n=1}^{\infty}$ is convergent, then prove that $\{S_n\}_{n=1}^{\infty}$ is bounded.

2) Define a non increasing sequence of real numbers.

For $n \in I$ let $t_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots + \frac{1}{n!}$ then prove that $\{t_n\}_{n=1}^{\infty}$ is monotone.

3) For $n \in I$ let $S_n = \frac{1 \cdot 3 \cdot 5 \cdot \ldots\ldots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdot \ldots\ldots \cdot 2n}$ then prove that $\{S_n\}_{n=1}^{\infty}$ is convergent.

Q. 5 Answer any TWO from the following. **[10]**

1) If $\{S_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers that diverges to infinity then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ and $\{S_n \cdot t_n\}_{n=1}^{\infty}$ also diverge to infinity.

2) Define convergent sequence of real numbers and prove that every convergent sequence is Cauchy sequence.

3) Let $\{S_n\}_{n=1}^{\infty}$ diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ converges then prove that $\{S_n + t_n\}_{n=1}^{\infty}$ diverges to infinity.

RAN-1043

RAN-1043

T.Y.B.Sc (Sem-V) Examination

March / April - 2019

Mathematics Paper : MTH-504 : Real Analysis - II

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

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Fill up strictly the details of ☛ signs on your answer book
Name of the Examination:
☛ T.Y.B.Sc (Sem-V)
Name of the Subject :
☛ Mathematics Paper : MTH-504 : Real Analysis - II
Subject Code No.: 1 0 4 3

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Figures to right indicate full marks of the corresponding questions.

(4) Follow usual notations.

1. Answer the following as directed (Any FIVE) **(10)**

(1) If $|x-3|<\frac{1}{10}$, then prove that $|x^2-x-6|<0.51$.

(2) If the real - valued functions f and g are continuous at $a \in R^1$, then prove that $f-g$ is also continuous at $a \in R^1$.

(3) Define: Metric for a Set & Equivalent Metrics.

(4) Justify: If ρ and σ are metrics for a set M, then $\rho-\sigma$ is also a metric for M.

(5) Justify: The sequence $\left\{x_n=\frac{1}{n}\right\}_{n=1}^{\infty}$ is not convergent sequence of points in the metric space R_d.

(6) Construct the open balls B[2018 ; 0.2018] and B[0.2018; 2018] in the metric space R_d.

(7) Give an illustration such that an infinite intersection of open sets in a metric space is open.

(8) Justify: $\left(0, \frac{1}{2}\right]$ is open in the metric space $<[0,1], |.|>$.

2. **Answer any TWO:** (

(1) Let f and g be the real - valued functions and $a, L, M \in R$. If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$, then prove that $\lim_{x \to a} [f(x) - g(x)] = L - M$.

(2) Define the continuity of a real-valued function at $a \in R^1$. If the real-valued functions f and g are continuous at $a \in R^1$, then prove that max $\{f, g\}$ is also continuous at $a \in R^1$.

(3) Prove that $<R, d>$ is a metric space, where the function $d : R \times R \to [0, \infty)$ is defined as : $d(x, x) = 0$; for every $x \in R$ and $d(x, y) = 1$; for $x \neq y$ in R.

3. **Answer any TWO:** (1

(1) Define a Cauchy Sequence in a metric space. If $\{x_n\}_{n=1}^{\infty}$ is a convergent sequence in the metric space Rd, then prove that there exists $N \in I$ such that $x_N = x_{N+1} = x_{N+2} = ...,$ Also, give an example of a Cauchy sequence in the metric space Rd.

(2) Prove that a sequence of points in a metric space $< M, \rho >$ can not converge to two distinct points of M.

(3) Prove that the metircs σ and τ for R^2 defined respectively as follows:

$\sigma(P, Q) = |x_1 - x_2| + |y_1 - y_2|$; $\tau(P, Q) = max\{|x_1 - x_2|, |y_1 - y_2|\}$;

Where $P = <x_1, y_1>$ & $Q = <x_2, y_2>$ in R^2;

are equivalent metrics for R^2.

4. **Answer any TWO:** (1

(1) Define an open ball about a point a in R^1. Prove that the real-valued function f is continuous at a in R^1 if and only if given $\in > 0$ there exists $\delta > 0$ such that

$f^{-1}\{B[f(a); \in]\} \supseteq B[a; \delta]$

(2) Let $f : <M_1, \rho_1> \rightarrow <M_2, \rho_2>$ and $g : <M_2, \rho_2> \rightarrow <M_3, \rho_3>$ be two functions. If f is continuous at $a \in M_1$ and g is continuous at $f(a) \in M_2$, then prove that gof is continuous at $a \in M_1$.

(3) Define an open ball in the metric space R_d. Prove that every function from R_d into any metric space is continuous on R_d.

5. **Answer any TWO:** **(10)**

(1) (i) Define an open set in a metric space.

(ii) Prove that : (a) Every singleton set is not open in R^1 ;

(b) Any singleton set is open in R_d.

(2) Prove that an arbitrary union of open sets in any metric space is open.

(3) Let G be an open subset of the metric space R_1 , then prove that x_G ; the characteristic function of G ; is continuous at each point of G.

RAN-1044

RAN-1044

B.Sc. Sem. V Examination

March / April - 2019

MH - 505 - Mathematics

(Graph Theory)

(Old or New to be mentioned where necessary)

[Total Marks: 50

સૂચના : / Instructions

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Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **B.Sc. Sem. V**

Name of the Subject :

☛ **MH - 505 - Mathematics**

Subject Code No.: 1 0 4 4

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Follow usual notations.
(3) Figures to the right indicate marks of the question.

Que:1 Answer any FIVE as directed. **[10]**

(1) Draw the graphs of the chemical compounds: C_2H_4 and N_20_3.

(2) What do you mean by isolated vertex and pendant vertex?

(3) In a graph G, if there is one and only one path between every pair of vertices, then prove that G is a tree.

(4) If G_1 and G_2 are vertex disjoint subgraphs, then what is $G_1 \oplus G_2$? Justify.

(5) Show that the number of pendent vertices in a binary tree is $(n + 1)/2$.

(6) If G is a simple graph with 15 vertices, then find the maximum number of edges in G, also write the maximum possible degree of a vertex of G.

(7) Let G be a graph with n vertices and e edges. If two vertices v_1 and v_2 of G are fused and replace by a single vertex v then what is the number of vertices and number of edges in G after fusion?

(8) Construct a regular graph of six vertices and a complete graph of five vertices.

Que:2 Attempt any TWO. **[10]**

(1) Define *finite graph*. Show that the sum of the degrees of all vertices in any finite graph G is a twice the number of edges in G.

(2) Explain seating arrangement problem for eleven persons.

(3) Explain the following terms using illustration:

Adjacent vertices and Adjacent edges, Parallel edges, Edges in series.

Que:3 Attempt any TWO. **[10]**

(1) Explain *isomorphism* of two graphs using illustration.

(2) Explain *union of graphs, intersection of graphs and ring sum of graphs* using illustration.

(3) Prove that a graph containing m edges can be decomposed in 2^{m-1} different ways into pair of subgraphs, g_1 and g_2.

Que:4 Attempt any TWO. **[10]**

(1) Show that a simple graph with n vertices and k components can have at the most $\frac{(n-k)(n-k+1)}{2}$ edges.

(2) Prove that a given connected graph G is an Euler graph if and only if all vertices of G are of even degree.

(3) Define a *complete graph*. Prove that a Hamiltonian circuit can be constructed in a complete graph.

Que:5 Attempt any TWO. **[10]**

(1) Show that a tree with n vertices has n - 1 edges.

(2) Show that the distance between vertices of a connected graph is a metric.

(3) Define *center, radius and diameter of a tree*. Using example, show that the radius of a tree need not be half its diameter.

RAN-1045

RAN-1045

T.Y.B.Sc. (Sem. V) Examination

March / April - 2019

Mathematics : Paper - 506

(Number Theory)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

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Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T.Y.B.Sc. (Sem. V)**

Name of the Subject :
☛ **Mathematics : Paper - 506**

Subject Code No.: 1 0 4 5

Seat No.:

Student's Signature

(1) Digits to the right indicates marks of the question.
(2) Follow the usual notations.

Q-1 Answer any five questions: **(10)**

(1) If a is an odd integer, Show that gcd $(3a, 3a + 2) = 1$.

(2) Show that any composite three digit number must have a prime factor less than or equal to 31 .

(3) Check whether 2093 is a prime or not? If not write its prime power factorization.

(4) Verify whether integers 4, 11, 12, 13, 22, 82, 91 form a CRS (mod 7)

(5) Find 1cm (306, 657).

(6) For any integer a prove that $a^4 \equiv 0$ *or* $1 \pmod 5$.

(7) Is $N = (447836)_{NINE}$ is divisible by 10.

(8) Solve: $18x + 81y = 26$.

Q-2 **Answer any two questions :** **(10**

(1) Given integers a and b , not both of them zero then, show that there exists integers x and y such that gcd $(a, b) = ax + by$.

(2) If gcd (a, b, c) = gcd (gcd$(a, b), c)$ = gcd$(a,$ gcd$(b, c))$ = gcd(gcd$(a, c), b)$ then find integers x,y,z such that gcd (198,288,512) = $198x + 288y + 512z$.

(3) For positive integers a and b, *prove that* gcd (a, b) *lcm* $(a,b) = ab$

Q-3 **Answer any two questions :** **(10**

(1) Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if d|c where d = gcd(a,b) *If* $x = x_0$, $y = y_0$ is a particular solution, then find all other solutions.

(2) Determine the all positive solutions of the Diophantine equation $221x + 35y = 11$.

(3) Prove that $1 + \sqrt{2}$ is an irrational number.

Q-4 **Answer any two questions :** **(10**

(1) If $a \equiv b \pmod{n}$ then prove that gcd (a, n) = gcd (b, n).

(2) If p_n is the n^{th} prime number, then prove that $p_n \leq 2^{2^{n-1}}$.

(3) Find the remainder when 8888^{8888} is divided by 9 .

Q-5 **Answer any two questions :** **(10**

(1) Let $N = a_m 9^m + a_{m-1} 9^{m-1} + a_{m-2} 9^{m-2} + ---- + a_1 9^1 + a_0$,

be the representation of the positive integer N, with $0 \leq a_k \leq 8$, and $T = a_0 + a_1 + a_2 + a_3 + ---- + a_m$. Then Prove that $8|N$ if and only if 8|T.

(2) Prove that the integer $1835^{1910} + 1986^{2061}$ is divisible by 7.

(3) Working modulo 9 or 11 find the missing digit x in the calculation $2x99561 = [3(523 + x)]^2$.

---

6) Let $o(G)$ = 31. Is G cyclic? Justify your answer.

7) If $o(G)$ = 24 and H is a subgroup of G such that $o(H)$ = 12. Is H a normal subgroup of G? Justify your answer.

8) Is every subgroup of $Z(G)$ a normal subgroup of G? Justify your answer.

Que 2: Answer any TWO of the following: (10)

1) If a, b are integers, not both 0, then prove that (a, b) exists. Moreover, show that there exists integers m_0 and n_0 such that $(a, b) = m_0 a + n_0 b$.

2) Prove that the relation "congruence modulo n" defines an equivalence relation on the set of integers.

3) To check that n is a prime number, prove that it is sufficient to show that it is not divisible by any prime number p, such that $p \leq \sqrt{n}$.

Que 3: Answer any TWO of the following: (10)

1) If H is any arbitrary non-empty subset of a group G then state and prove the necessary and sufficient condition for H to be a subgroup of G.

2) Let G be the set of all symbols a_i ; i = 0, 1, 2, ..., 6 where

   $a_i \cdot a_j = ai + j$; if $i + j < 7$ and $a_i \cdot a_j = a_{i+j-7}$ if $i + j \geq 7$.

   Prove that G is an abelian group under the given operation.

3) In a group G, if $a, b \in G$ are any elements and $(ab)^n = a^n b^n$ for three consecutive integers n, then prove that G is abelian.

Que 4: Answer any TWO of the following: (10)

1) State and prove Lagrange's Theorem.

2) Prove that every subgroup of cyclic group is cyclic.

3) Let H be a subgroup of a group G and $a \in G$. If

   $aHa^{-1} = \{aha^{-1} \mid h \in H\}$, then show that aHa^{-1} is a subgroup of G. Also find $o(aHa^{-1})$, if H is finite.

Que 5: Answer any TWO of the following: (10)

1) State and prove fundamental theorem of homomorphism.

2) Let G be the group of integers under addition. Let N be the subgroup of G which contains all the multiples of 5. Find all elements of quotient group G/N. Also find index of N in G.

3) Define: Normal subgroup. Prove that a subgroup N is a normal subgroup of a group G if and only if $gNg^{-1} = N$; for every $g \in G$.

---

2. **Answer the following (Any two).** **[10**

1 Prove that : The Set $\mathbb{R}_3$ of all three tuples of real numbers is a real vector space.

2. Determine which of the following set is subspace of V_3.

   (a) $\{(x_1, x_2, x_3) \in V_3 / x_1 = 2x_2 \text{ } or \text{ } x_3 = 3x_2\}$

   (b) $\{(x_1, x_2, x_3) \in V_3 / x_1 \cdot x_2 = 0\}$

3. Prove that the set $\{\alpha_1 u_1 + \alpha_2 u_2 + \alpha_3 u_3 ++++ \alpha_n u_n / \alpha_i \in R, u_i \in V, i = 1 \text{ } to \text{ } n$ be a subspace of a vector space V.

3. **Answer the following (Any two).** **(10)**

1) If U and W are two subspaces of a vector space V, then prove that $U + W$ is a subspace of V and $U + W = [U \cup W]$.

2) Let U and W be two subspaces of vector space V and $Z = U + W$. Then prove that $Z = U \oplus W$ if and only if any vector $z \in Z$ can be expressed uniquely as the form $z = u + w, u \in U, w \in W$.

3) If U and W are two subspaces of a vector space V then prove that $U + W = U$ if and only if $W \subset U$.

4. **Answer the following (Any two).** **(10)**

1) In a vector space V, prove that

   a) If v *is* a trivial linear combination of $v_1, v_{2,,,,,,}v_n$ then the set $\{v, v_1, v_{2,,,,,,,} v_n\}$ is L.D.

   b) If u, v and ware linearly independent vectors then the vectors $u + v$, $v + w$, $u + w$ are also L.I vectors.

2) (a) If a set is L.I then prove that any subset of it is also L.I.

   (b) Show that θ is collinear with any non zero vector v.

3) Prove that : In a vector space V, suppose $\{v_1, v_2, \dots\dots\dots\dots, v_n\}$ is an ordered set of vectors with $v_1 \neq \theta$. The set is L.D if and only if one of the vectors $v_2, v_3, \dots\dots\dots, v_n$ say v_k, belongs to the span of $v_1, v_2, \dots.v_{k-1}$.

5. **Answer the following (Any two).** **(10)**

1) Let $\{u_1, u_2, u_3, \ldots\ldots\ldots, u_n\}$ be the set of n L.I vectors in an n^{th} dimensional vectors space V then prove that the set $\{u_1, u_2, u_3, \ldots\ldots\ldots u_n\}$ is basis of V.

2) In a vector space V, the set $B = \{v_1, v_2, ,,,,,,,,, v_n\}$ generates V. Prove that the expression $v = \alpha_1 v_2 + \alpha_2 v_2 + - - - - - + \alpha_n v_n$ is unique for every $v \in V$ if and only if the set is B *is* L.I.

3) Define : Basis in a vector space V. Prove that the subset. B={(1,1,1,), (1,-1,1), (0,1,1)} form a basis for V_3.

RAN-1046

RAN-1046

T.Y.B.Sc. (Sem-V) Examination

March / April - 2019

Operation Research-I

(Mathematics-Elective Generic-5001)(3)

[Total Marks: 50

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. **Fill up strictly the details of ☛ signs on your answer book** Name of the Examination: ☛ **T.Y.B.Sc. (Sem-V)** Name of the Subject : ☛ **Operation Research-I** Subject Code No.: 1 0 4 6	Seat No.: Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of the corresponding questions.

Q:1 (a) Attempt any three: **06**

1. Write the canonical form of the L.P.P.
2. Define: feasible solution and bounded solution of L.P.P.
3. Define slack variables by giving proper illustration.
4. Write any two relationship between the primal L.P.P. and the dual L.P.P.
5. When the basic feasible solution becomes degenerate or non-degenerate?

(b) Write the dual of the following LPP. (Any One)

1. Min $Z = x_1 + 2x_2$

   Subject to $2x_1 + 4x_2 \leq 160$

   $x_1 - x_2 = 30$

   $x_1 \geq 10$

   And $x_1, x_2 \geq 0$

2. Max $Z = x_1 - x_2 + x_3$

   Subject to $x_1 + x_2 + x_3 \leq 10$

   $2x_1 - x_3 \leq 2$

   $2x_1 - 2x_2 - 3x_3 \leq 6$

   And $x_1, x_2, x_3 \geq 0$

Q:2 Attempt any two:

1. Use graphical method the following L.P.P.

   Max $Z = 10\, x_1 + 15\, x_2$

   Subject to $2x_1 + x_2 \leq 26$

   $2x_1 + 4x_2 \leq 56$

   $-x_1 + x_2 \leq 5$

   And $x_1, x_2 \geq 0$

2. Solve the following LPP by using simplex method.

   Maximize $Z = x_1 - 3x_2 + 2x_3$

   Subject to $3x_1 - x_2 + 3x_3 \leq 7$

   $-x_1 + 2x_2 \leq 6$

   $-4x_1 + 3x_2 + 8x_3 \leq 10$

   And $x_1, x_2, x_3 \geq 0$

3. Solve the following LPP by Big M-method.

   Max $Z = 10x_1 + 15x_2$

   s.t. $2x_1 + x_2 \leq 26$

   $2x_1 + 4x_2 \leq 56$

   $-x_1 + x_2 \leq 5$

   And $x_1, x_2 \geq 0$

Q:3 Attempt any two: **20**

1. Solve the following LPP by using two phase simple method.

Minimize $Z = \frac{15}{2}x_1 - 3x_2$

Subject to $3x_1 - x_2 - x_3 \geq 3$

$x_1 - x_2 + x_3 \geq 2$

And $x_1, x_2, x_3 \geq 0$

2. Solve the following LPP by using two phase simplex method.

Min Z = $x_1 + x_2$

Subject to $2x_1 + 4x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

And $x_1, x_2 \geq 0$

3. Solve the following LPP by using Big M-method.

Minimize $Z = 3x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \geq 4$

And $x_1, x_2 \geq 0$
