RAN-0838

B

RAN-0838

S.Y.B.Sc. (Mathematics) (Sem. III) Examination

March / April - 2019

Mathematics : MTH - 301 (New)

(Advanced Calculus-I)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **S.Y.B.Sc. (Mathematics) (Sem. III)**

Name of the Subject :
☛ **Mathematics : MTH - 301 (New)**

Subject Code No.: 0 8 3 8

Seat No.:

Student's Signature

(2) The question paper has Four sections and 18 questions in all.
(3) All sections and questions are compulsory.
(4) Follow usual symbols.
(5) Use of non-programmable calculator is allowed.
(6) These are to be answered by writing the correct option in your answer sheet.

SECTION - A : Q.1 to 4 Multiple Choice Questions : (1 mark)
SECTION - B : Q. 5 to 8 Multiple Choice Questions : (2 marks)
SECTION - C: Q. 9 to 14 Multiple Choice Questions : (3 marks)
SECTION - D: Q. 15 to 18 Multiple Choice Questions : (5 marks)

O.M.R. Sheet ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ O.M.R. Sheetની પાછળ છાપેલ છે.
Important instructions to fillup O.M.R. Sheet are given on back side of the provided O.M.R. Sheet.

Section -A

Q: 1 div grad f = ______.

(A) $\nabla^2 f$

(B) ∇f

(C) $\nabla . \Delta$

(D) ∇f^2

Q: 2 $\iiint_v \nabla . \vec{F}\, dv$ = ______

(A) $\iint_S \vec{F}\, dv$

(B) $\iint_S \vec{F} . \hat{n}\, ds$

(C) $\int_S \vec{F} . \hat{n}\, ds$

(D) $\iint_C \vec{F} . \hat{n}\, ds$

Q: 3 If $(x, y) = \log\sqrt{x^2 + y^2}$ then F_x = ______.

(A) $-\dfrac{x}{(x^2 + y^2)}$

(B) $\dfrac{x}{(x^2 + y^2)}$

(C) $\dfrac{x^2}{(x^2 + y^2)}$

(D) $\dfrac{x^2}{(x + y)^2}$

Q: 4 If $y_1(x_1 - x_2) = 0$ and $y_2\,(x_1^2 + x_1 x_2 + x_2^2) = 0$ then $\dfrac{\partial(F_1, F_2)}{\partial(x_1, x_2)}$ = ______.

(A) $-y_1 y_2\,(3x_1 + 3x_2)$

(B) $y_1 - y_2\,(3x_1 + 3x_2)$

(C) $y_1 y_2 (3x_1 + x_2)$

(D) $y_1 y_2 (3x_1 + 3x_2)$

Section -B

Q: 5 If the curve C is a circle $x^2 + y^2 = a^2$ then $\int_c (x\, dy - y\, dx)$ = ______ .

(A) $-2\pi a^2$

(B) 2π

(C) $2\pi a^2$

(D) $\pm 2\pi a^2$

Q: 6 $\lim_{(x,y) \to (0,0)} \dfrac{\tan(x + y)}{x + y}$ = ______ .

(A) 0

(B) 1

(C) –1

(D) ∞

Q: 7 If $x = p\cos\phi$, $y = p\sin\phi$ and $z = z$ then $\dfrac{\partial(x, y, z)}{\partial(p, \phi\, z)} = $ ______ .

(A) p

(B) $-p$

(C) ϕ

(D) z

Q: 8 If $\vec{F} = xy^2\hat{i} + 2x^2y\hat{j} - 3yz^2\hat{k}$ then $div\ \vec{F}$ at point $(1, -1, 1)$ is ______ .

(A) -9

(B) 9

(C) 6

(D) -6

Section -C

Q: 9 If C is the boundary of the ellipse $x^2 + 4y^2 = 4$ then

$\oint_c (2x - y)\, dx + (x + 3y)\, dy = $ ______.

(A) -2π

(B) -4π

(C) 4π

(D) 2π

Q: 10 If $z = xy^2 + x^2y$ and $x = at^2$, $y = 2at$ $(a \neq 0)$ then $\dfrac{dz}{dt} = $ ______.

(A) $a^3[16t^3 + 10t^4]$

(B) $-a^3[16t^2 + 10t^3]$

(C) $a^3[16t^2 + 10t^2]$

(D) $a^3[16t^3 = 10t^4]$

Q: 11 $\nabla \cdot (\nabla \times \vec{F}) = $ ______.

(A) ∇

(B) -1

(C) 0

(D) 1

Q: 12 If $u = \sin^{-1}\left(\dfrac{x^2 + y^2}{x + y}\right)$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = $ ______.

(A) $\sin u$

(B) $-\sin 2u$

(C) $-\sin u$

(D) $\sin 2u$

Q: 13 If $x^2 + y^2 = r^2$ and $\tan\theta = \frac{y}{x}$ then $\frac{\partial(r,\theta)}{\partial(x,y)} =$ ______.

(A) r

(B) $\frac{1}{r}$

(C) $-r$

(D) $-\frac{1}{r}$

Q: 14 If $r = |\vec{r}|$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then $\nabla r^n =$ ______.

(A) $n \cdot r^{n-2}\vec{r}$

(B) $n \cdot r^{n-1}\vec{r}$

(C) $n \cdot r^{n-2}$

(D) $n^2 \cdot r^{n-1}\vec{r}$

Section -D

Q: 15 If $\vec{u}(t) = 5t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ then $\int_1^2 \left(\vec{u} \times \frac{d^2\vec{u}}{dt^2}\right)dt =$ ______.

(A) $28\hat{i} + 75\hat{j} - 30\hat{k}$

(B) $-28\hat{i} + 75\hat{j} - 30\hat{k}$

(C) $-28\hat{i} - 75\hat{j} + 30\hat{k}$

(D) $28\hat{i} - 75\hat{j} + 30\hat{k}$

Q: 16 If $u = cosec^{-1}\left(\sqrt{\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}}}\right)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$ ______.

(A) $\frac{1}{12}\tan u$

(B) $-\frac{1}{12}\tan u$

(C) $\tan u$

(D) $\frac{1}{12}\tan^2 u$

Q: 17 Expand $F(x,y) = \frac{y^2}{x^3}$ into series expansion form up to the second order term in the powers of $(x-1)$ and $(y+1)$.

(A) $1 + [3(x-1) + 2(y+1)] + [6(x-1)^2 + 6(x-1)(y+1) + (y+1)^2]$

(B) $1 - [3(x+1) + (y+1)] + [6(x-1)^2 + 6(x-1)(y+1) + (y+1)^2]$

(C) $1 - [3(x+1) + 2(y+1)] + [6(x-1)^2 + (x-1)(y+1) + (y+1)^2]$

(D) $1 - [3(x-1) + 2(y+1)] + [6(x-1)^2 + 6(x-1)(y+1) + (y+1)^2]$

Q: 18 If $\vec{\phi} = A(x,y,z)\hat{i} + B(x,y,z)\hat{j} + C(x,y,z)\hat{k}$ then $curl\ \vec{\phi} =$ ______.

(where, $\vec{\phi}_x = \frac{\partial}{\partial_x}(\phi)$)

(A) $\hat{i}\times\vec{\phi}_x + \hat{j}\times\vec{\phi}_y + \hat{k}\times\vec{\phi}_z$

(B) $\hat{i}\times\vec{\phi}_x - \hat{j}\times\vec{\phi}_y + \hat{k}\times\vec{\phi}_z$

(C) $\hat{i}\times\vec{\phi}_x + \hat{j}\times\vec{\phi}_y - \hat{k}\times\vec{\phi}_z$

(D) $\hat{i}\cdot\vec{\phi}_x + \hat{j}\cdot\vec{\phi}_y + \hat{k}\cdot\vec{\phi}_z$

RAN-0839

B

RAN-0839

S.Y.B.Sc. (Sem-III) Examination

March / April - 2019

Mathematics-MTH-302

(Numerical Analysis-I)

(New Course)

[Total Marks: 50

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S.Y.B.Sc. (Sem-III)**

Name of the Subject :

☛ **Mathematics-MTH-302**

Subject Code No.: 0 8 3 9

Seat No.:

Student's Signature

(2) The question paper consists of Four sections and 18 questions in all.

(3) All the sections and questions are compulsory.

(4) Follow usual notations.

(5) Use of non-programmable calculator is allowed.

(6) Figures to the right indicate marks of the questions.

(7) There are to be answered by writing the correct option in your answer sheet.

O.M.R. Sheet ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ O.M.R. Sheetની પાછળ છાપેલ છે.

Important instructions to fillup O.M.R. Sheet are given on back side of the provided O.M.R. Sheet.

Section A
(Each question carries ONE mark)

1] ∇y_{n-2} = ________

(A) Δy_{n-3} (B) Δy_{n-1} (C) δy_{n-1} (D) δy_{n-3}

2] If (1, 2·5), (2, 3·7) and (3, 4·9), then $\Delta^2 y_0$ = ________

(A) 0 (B) 1 (C) 2 (D) 3

3] The relative error = ________

(A) $\dfrac{\textit{Absolute Error}}{\textit{True Value}}$ (B) $\dfrac{\textit{Error}}{\textit{True Value}}$

(C) $\dfrac{\textit{Percentage Error}}{\textit{True Value}}$ (D) None of these

4] The root of the equation $e^x + x^3 = 0$ lies between ________

(A) –2 and –1 (B) 0 and 1 (C) – 3 and – 2 (D) –1 and 0

Section B
(Each question carries TWO marks)

5] $\Delta\left[y_k^2\right]$ = ________

(A) $(y_k + y_{k+1})\Delta y_k$ (B) $(y_k - y_{k+1})\Delta y_k$

(C) $(y_{k-1} + y_k)\Delta y_k$ (D) None of these

6] The relative error of the number 8.6, if both of its digits are correct, is ________

(A) 0.00058 (B) 0.0058 (C) 0.058 (D) 0.58

7] Using false-position method, a root of the equation $x^3 - 2x - 9 = 0$ between 2 and 3 correct to two decimal places is ________

(A) 2.19 (B) 2.29 (C) 2.39 (D) 2.49

8] If y(1) = 2.5, y(1.5) = 3.4, y(2) = 4.2 and y(2.5) = 5.0, then the value of $\Delta^3 y_0$ is ________

(A) 0 (B) 0.1 (C) 0.2 (D) 0.3

Section C

(Each question carries THREE marks)

9] The product of the numbers 2.5 and 48.289 (both of which being correct to the significant figures given) is ________

(A) 12×10^2 (B) 1.2×10^2 (C) 0.12×10^2 (D) None of these

10] If (0, 102), (0.1, 205), (0.2, 415), (0.3, 604) and (0.4, 814), then $\Delta^4 y_{-2}$ = ________

(A) 42 (B) – 128 (C) 170 (D) None of these

11] Using method of false-position, the real root of the equation $x^3 - 9x + 1 = 0$ correct up to two decimal places is ________

(A) 2.91 (B) 2.94 (C) 2.98 (D) 2.80

12] The relative error in the sum of the numbers 2.665, 3.321 and 5.143 to four significant digits is ________

(A) 0.0002 (B) 0.0001 (C) 0.0003 (D) 0.0004

13] Using Newton Raphson method, the real root of the equation $\sin x + 3 = 2x$ correct up to three decimal places is ________

(A) 2.095 (B) 1.592 (C) 2.895 (D) 1.962

14] $\mu^2 \equiv$ ________

(A) $\delta^2 + 1$ (B) $\frac{\delta^2}{2} + 1$ (C) $\frac{\delta^2}{3} + 1$ (D) $\frac{\delta^2}{4} + 1$

Section D

(Each question carries FIVE marks)

15] The table gives the value of $f(x)$, then, $f(40)$ = ________

x	10	30	50	70
$y = f(x)$	2.707	3.027	3.386	3.794

(Using Guass's Backward Difference Interpolation Formula)

(A) 2.002 (B) 2.201 (C) 3.002 (D) 3.201

16] Using iteration method, the real root of the equation $x^3 - 2x - 7 = 0$ correct up to three decimal places if ________

(A) 2.258 (B) 2.598 (C) 2.298 (D) 2.128

17] The absolute error in the sum of the numbers 105.6, 27.28, 5.63, 0.1467, 0.000523, 208.5, 0.0235, 0.432 and 0.0467 is ________ . (Where each number is correct to the digits given.)

(A) 347.7 ± 0.1

(B) 347.7 ± 0.2

(C) 347.7 ± 0.25

(D) None of these

18] The table gives the value of $y(x)$, then y (3.75) = ________

x	1.5	2.0	2.5	3.0	3.5	4.0
$y(x)$	25.88	34.20	42.26	50.00	57.36	64.28

(A) 60.87875

(B) 60.8704

(C) 60.8700

(D) None of these

RAN-0840

B

RAN-0840

Second Year B.Sc. (Sem. III) Examination

March / April - 2019

MTH - 303 : Mathematics

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **Second Year B.Sc. (Sem. III)**

Name of the Subject :
☛ **Mathematics**

Subject Code No.: 0 8 4 0

Seat No.:

Student's Signature

1) There are FOUR sections (A, B, C, D) in this question paper.
2) There is only ONE correct answer for each question.
3) Follow the usual symbols.

Section - A : Q. 1 to 4 Multiple choice questions : (1 mark)
Section - B : Q. 5 to 8 Multiple choice questions : (2 marks)
Section - C : Q. 9 to 14 Multiple choice questions : (3 marks)
Section - D : Q. 15 to 18 Multiple choice questions : (1 mark)

O.M.R. Sheet ભરવા અંગેની અગત્યની સૂચનાઓ આપેલ O.M.R. Sheetની પાછળ છાપેલ છે.
Important instructions to fillup O.M.R. Sheet are given on back side of the provided O.M.R. Sheet.

1. General solution $X^2 \frac{d^2y}{dx^2} + y = 0$ is 1

(A) $y = 2\sqrt{x}\cos\left(\frac{3}{2}\log x + \alpha\right)$

(B) $y = \sqrt{x}\cos\left(\frac{\sqrt{3}}{2}\right)\log x + \alpha$

(C) $y = 2\sqrt{x}\cos\left(\frac{\sqrt{3}}{2}\log x + \alpha\right)$

(D) $y = 2x\cos\left(\frac{\sqrt{3}}{2}\log x + \alpha\right)$

2. A partial differential equation by eliminating F from $z = F(x^2 + y^2)$ is 1

(A) $yp + xq = 0$

(B) $yp - xq = 0$

(C) $xp - yq = 0$

(D) $xp = yq = 0$

3. A partial differential equation by eliminating a *and* b from $z = axe^y + \frac{1}{2}a^2e^{2y} + b$ is 1

(A) $q = px + p^2$

(B) $q = p + p^2y$

(C) $p = qx + q^2$

(D) $q = py + p^2$

4. Solution of partial differential equation $pq = xy$ is 1

(A) $z = \frac{1}{2}[a^2x^2 + y^2 + 2ab]$

(B) $z = \frac{1}{2a}[a^2x^2 + ay^2 + b]$

(C) $z = \frac{1}{2a}[a^2x^2 + y^2 + ab]$

(C) $z = \frac{1}{2a}[a^2x^2 + y^2 + 2ab]$

5. General Solution of partial differential equation $yzp + zxq = xy$ is 2

(A) $f(x^2 + y^2, y^2 - z^2) = 0$

(B) $f(x^2 - y^2, y^2 - z^2) = 0$

(C) $f(x^2 - y^2, y^2 + z^2) = 0$

(D) $f(x^2 + y^2, y^2 + z^2) = 0$

6. Particular Integral of a differential equation 2

$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x^2\log x$ is

(A) $\frac{x^3}{6}(\log x)^3$

(B) $\frac{x^2}{6}(\log x)^2$

(C) $-\frac{x^2}{6}(\log x)^3$

(D) $\frac{x^2}{6}(\log x)^3$

7. A partial differential equation by eliminating f and g from $z = f(x + ay) + g(x - ay)$ is 2

(A) $a^2 r - t = 0$

(B) $a^2 r + t = 0$

(C) $a^2 s - t = 0$

(D) $a^2 s + t = 0$

8. Complete solution of partial differential equation $q = 3p^2$ is **2**

(A) $z = ax + a^2y + a^3$
(B) $z = a^2x + ay + b$
(C) $z = ax + 3a^2y + b$
(D) $z = ax - a^2y + b$

9. The general solution of $x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} - 2y = 0$ is **3**

(A) $y = Ax^{-2} + x^{\frac{5}{2}}\left[Bx^{\frac{\sqrt{21}}{2}} + Cx^{\frac{-\sqrt{21}}{2}}\right]$

(B) $y = Ax^{2} + x^{\frac{5}{2}}\left[Bx^{\frac{\sqrt{21}}{2}} + Cx^{\frac{-\sqrt{21}}{2}}\right]$

(C) $y = Ax^{2} + x^{\frac{-5}{2}}\left[Bx^{\frac{\sqrt{21}}{2}} + Cx^{\frac{-\sqrt{21}}{2}}\right]$

(D) $y = Ax^{-2} + x^{\frac{-5}{2}}\left[Bx^{\frac{\sqrt{21}}{2}} + Cx^{\frac{-\sqrt{21}}{2}}\right]$

10. The general solution of $[D^2 + 4xD + 4x^2]\, y = 0$ where $D \equiv \frac{d}{dx}$ by removal of first order derivative is **3**

(A) $y = e^{x^2}[c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x}]$
(B) $y = e^{x}[c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x}]$
(C) $y = e^{-x^2}[c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x}]$
(D) $y = e^{-x}[c_1e^{\sqrt{2}x} + c_2e^{-\sqrt{2}x}]$

11. The general solution of $[D^2 + \cot x\, D + 4\operatorname{cosec}^2 x]y = 0$, where $D \equiv \frac{d}{dx}$ by transforming the independent variable x to z is **3**

(A) $c_1 \cos\left[\log \tan\left(\frac{x}{2}\right)\right] + c_2 \sin\left[\log \tan\left(\frac{x}{2}\right)\right]$

(B) $c_1 \cos[2\log \tan(x)] + c_2 \sin[2\log \tan(x)]$

(C) $c_1 \cos\left[2\log \tan\left(\frac{x}{2}\right)\right] + c_2 \sin\left[2\log \tan\left(\frac{x}{2}\right)\right]$

(D) $c_1 \cos\left[\frac{1}{2}\log \tan\left(\frac{x}{2}\right)\right] + c_2 \sin\left[\frac{1}{2}\log \tan\left(\frac{x}{2}\right)\right]$

12. Complete solution of partial differential equation $p + 3q = 5z + \tan(y - 3x)$ is **3**

(A) $f(y - x,\ e^{-5x}(5z + \tan(y - x))) = 0$
(B) $f(y + 3x,\ e^{-5x}(5z + \tan(y - 3x))) = 0$
(C) $f(y - 3x,\ e^{-5x}(5z + \tan(y + 3x))) = 0$
(D) $f(y - 3x,\ e^{-5x}(5z + \tan(y - 3x))) = 0$

13. Complete solution of partial differential equation $z^2(p^2 + q^2 + 1) = c^2$ is 3

(A) $(ax + y + b)^2 = (a^2 - 1)(c^2 - z^2)$

(B) $(ax + y + b)^2 = (a^2 + 1)(c^2 + z^2)$

(C) $(x + ay + b)^3 = (a^2 + 1)(c^2 - z^2)$

(D) $(x + ay + b)^2 = (a^2 + 1)(c^2 - z^2)$

14. Complete solution of partial differential equation $p = qy + q^2$ is 3

(A) $z = \frac{1}{4}[-y^2 + y\sqrt{y^2 + 4a} + 4a\,log\,(y + \sqrt{y^2 + 4a})] + ax + b$

(B) $z = \frac{1}{4}[y^2 + y\sqrt{y^2 + 4a} + 4alog\,(y + \sqrt{y^2 + 4a})] + ax + b$

(C) $z = \frac{1}{4}[-y^2 + y\sqrt{y^2 - 4a} + 4alog\,(y + \sqrt{y^2 + 4a})] + ax + b$

(D) $z = \frac{1}{4}[-y^2 + y\sqrt{y^2 + 4a} + a\,log\,(y + \sqrt{y^2 + 4a})] + ax + b$

15. The general solution of $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ is 5

(A) $Ax^{-1} + Bx^{-2} + x^2e^x$

(B) $Ax^{-1} + Bx^2 + x^{-2}e^x$

(C) $Ax^{-1} + Bx^{-2} + x^{-2}e^x$

(D) $Ax^1 + Bx^{-2} + x^{-2}e^x$

16. The complete Integral of partial differential equation $p^2z^4 + q^2z^2 = 1$ is 5

(A) $(z^2 - a^2)^3 = 9(x+ay+b)^2$

(B) $(z^2+a^2)^{-3} = 9(x+ay+b)^2$

(C) $(z^2 + a^2)^3 = 9(x+ay+b)^{-2}$

(D) $(z^2+a^2)^3 = 9(x+ay+b)^2$

17. The general solution of $(1 + 2x)^2\frac{d^2y}{dx^2} - 6(1 + 2x)\frac{dy}{dx} + 16y = 8(1 + 2x)^2$ is 5

(A) $y = (1 + 2x)^2[A + B\,log\,(1 + 2x) + \{log(1 + 2x)\}^2]$

(B) $y = (1 + 2x)^{-2}[A + log(1 + 2x) + B\,log(1 + 2x)^2]$

(C) $y = (1 + 2x)^2[A + Blog(1 + 2x) + Clog(1 + 2x)^2]$

(D) $y = (1 + 2x)[A + Blog(1 + 2x) + log(1 + 2x)^2]$

18. The general solution of $4x^2\frac{d^2y}{dx^2}+4x^5\frac{dy}{dx}+(x^8+6x^4+4)\,y=0$ by removal of first order derivative is 5

(A) $y=e^{\frac{1}{8}x^4}x^{\frac{1}{2}}\left[A\cos\left(\frac{\sqrt{3}}{2}\log x\right)+B\sin\left(\frac{\sqrt{3}}{2}\log x\right)\right]$

(B) $y=e^{-\frac{1}{8}x^4}x^{\frac{1}{2}}\left[A\cos\left(\frac{\sqrt{3}}{2}\log x\right)+B\sin\left(\frac{\sqrt{3}}{2}\log x\right)\right]$

(C) $y=e^{-\frac{1}{8}x^4}x^{\frac{1}{2}}\left[A\cos\left(\frac{\sqrt{3}}{2}x\right)-B\sin\left(\frac{\sqrt{3}}{2}x\right)\right]$

(D) $y=e^{-\frac{1}{8}x^4}x^{\frac{1}{2}}\left[A\cos\left(\frac{\sqrt{3}}{2}x\right)+B\sin\left(\frac{\sqrt{3}}{2}x\right)\right]$

RAN-0950

RAN-0950

S.Y.B.Sc. (Mathematics) (Sem IV) Examination

March / April - 2019

Paper-401 Advanced Calculus-II (Mathematics) (New)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **S.Y.B.Sc. (Mathematics) (Sem IV)**

Name of the Subject :
☛ **Paper-401 Advanced Calculus-II (Mathematics) (New)**

Subject Code No.: 0 9 5 0

Seat No.:

Student's Signature

(1) Figures to the right indicate marks of the question.
(2) Follow usual notations and conventions.

Q. 1 **Answer any Five from the following.** **[10]**

1. Define local minimum value for a bivariate function.
2. Find f_{xx} for the function $f(x, y) = \sin x \sin y \sin (x+y)$ where $0 \le x, y \le \frac{x}{2}$.
3. Evaluate : $\int_0^2 \int_0^{\frac{y}{2}} y \, dydx$
4. Evaluate : $\int_0^x \int_0^{\sin x} y \, dxdy$
5. Evaluate : $\int_0^\infty x^4 e^{-x} dx$
6. Define Γn and evaluate $\Gamma 1$.

7. Find $L^{-1}\left[\dfrac{3p}{p^2+16}\right]$.

8. Find $L[t^7]$.

Q. 2 **Attempt any Two.** [10]

(a) Find three positive numbers whose sum is 30 and the product is maximum.

(b) Discuss about the extreme points of the bivariate function $f(x, y) = x^3 y^2 (1 - x - y)$.

(c) Find the point $P(x, y)$ from which the sum of the distances from the axes and the line $x + y = 8$ is minimum.

Q. 3 **Attempt any Two.** [10]

(a) In usual notations prove that $B(l, m) = \dfrac{\Gamma l \, \Gamma m}{\Gamma(l+m)}$.

(b) Define the Beta integral and prove that $B(l,m) = \int_0^\infty \dfrac{y^{(m-1)}}{(1+y)^{(l+m)}} dy$.

(c) Prove that :

(i) $\int_0^\infty x^6 e^{-2x} dx = \dfrac{45}{8}$

(ii) $\int_0^1 \dfrac{x \, dx}{\sqrt{1-x^5}} = \dfrac{1}{5} B\left(\dfrac{2}{5}, \dfrac{1}{2}\right)$

Q. 4 **Attempt any Two.** [10]

(a) Change the order of the double integral $\int_0^2 \int_{\frac{x^2}{4}}^{3-x} f(x,y) \, dxdy$.

(b) Find area enclosed between the curves $y^2 = 2x$ and $x^2 = 2y$ using double integral.

(c) Evaluae $\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 y^2 \, dxdy$

Q. 5 **Attempt any Two.** [10]

(a) If $L^{-1}[f(p)] = F(t)$ then prove that $L^{-1}[f(mp-n)] = \dfrac{1}{m} e^{\frac{nt}{m}} F\left(\dfrac{t}{m}\right); m > 0$.

(b) Prove that $L[F(t)] = \dfrac{1 + (p-1)e^{-4p}}{p^2}$; where $F(t) = \begin{cases} t & , 0 < t < 4 \\ 5 & , t > 4 \end{cases}$

(c) Evaluate :

(i) $L[2t^3 + 5e^{-t}]$

(ii) $L^{-1}\left[\dfrac{1}{(p+1)(p-2)}\right]$

RAN-7027

RAN-7027

B.Sc. (Sem. IV) Examination

March / April - 2019

Mathematics : MTH - 402
(Partial Differential Equations)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **B.Sc. (Sem. IV)**

Name of the Subject :
☛ **Mathematics : MTH - 402 (Partial Differential Equations)**

Subject Code No.: 7 0 2 7

Seat No.:

Student's Signature

(1) Digits to the right indicates marks of the question.
(2) Follow the usual notations.

Q-1. Answer the following : (any FIVE) **10**

(1) Form a partial differential equation by eliminating arbitrary constants a and b from $z = axe^y + \frac{1}{2}a^2e^{2y} + b$.

(2) Solve : $x(y-z)\,p + y(z-x)\,q = z(x-y)$.

(3) Find the complete integral of $p^3 + q^3 = 27$.

(4) Eliminate arbitrary function f from $z = f\left(\frac{x}{y}\right)$.

(5) Find C.F. of $2r + 5s + 2t = xy$.

(6) Find P.I. of $(D^2 - 2DD' + D'^2)\,z = e^{x-2y}$.

(7) Solve : $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} - 15\frac{\partial^2 z}{\partial y^2} = 0$

(8) Solve : $(D^2 + DD' + D' - 1)\,z = 0$

Q-2. **Answer the following : (any TWO)** **10**

(1) Obtain the partial differential equation by eliminating arbitrary Function φ from $\varphi(x+y+z,\ x^2+y^2+z^2)=0$.

(2) Solve : $(y^2+z^2)\,p - xyq + zx = 0$.

(3) Solve : $x^2(y-z)\,p + y^2(z-x)\,q = x^2(x-y)$.

Q-3. **Answer the following : (any TWO)** **10**

(1) Explain the method to solve the partial differential equation $F(z, p, q) = 0$

(2) Solve : $q = px + p^2$

(3) Solve : $(p^2+q^2)\,x = pz$. (by Char pit's Method)

Q-4. **Answer the following : (any TWO)** **10**

(1) Find the C.F. of $(D^2 + k_1 DD' + k_2 D'^2)\,z = f(x,y)$; $D \equiv \frac{\partial}{\partial x}$, $D' \equiv \frac{\partial}{\partial x}$, and k_1; k_2 are constant and roots of an auxiliary equation are real and equal.

(2) Solve : $\frac{\partial^2 z}{\partial x^2} + (a+b)\frac{\partial^2 z}{\partial x \partial y} + ab\frac{\partial^2 z}{\partial y^2} = xy$.

(3) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = y\cos x$.

Q-5. **Answer the following : (any TWO)** **(10)**

(1) Show that the C.F. of $f(D, D')\,z = F(x, y)$ is given by $\phi_1(y+mx) + e^{cx}\phi_2(y+mx)$, $f(D,D') = (D-mD')(D-mD'-c)$

(2) Solve : $[D^2 - DD' - 2D'^2 + 2D + 2D']z = e^{2x+3y}$

(3) Solve : $(2DD' + D'^2 - 3D')\,z = 5\cos(3x-2y)$

RAN-0951

RAN-0951

B.Sc. (Sem.-IV) Examination

March / April - 2019

Mathematics -MTH - 402

Numerical Analysis-II (New Course)

[Total Marks: 50

સૂચના : / Instructions

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **B.Sc. (Sem.-IV)**

Name of the Subject :
☛ **Mathematics -MTH - 402**

Subject Code No.: 0 9 5 1

Seat No.:

Student's Signature

(2) All questions are compulsory.

(3) Follow usual notations.

(4) Figures to the right indicate total marks of the question.

(5) Use of Scientific non-programmable calculator is allowed.

Que:1 Answer any FIVE as directed. **[10]**

(1) Use divided difference interpolation formula to obtain the function $y(x)$ for the given data (1, –5) and (2, –3).

(2) If $y(x) = \frac{1}{x}$, then find the value of $[x_1, x_2, x_3]$.

(3) Prove that $[x, y] = [y, x]$.

(4) Write the Lagrange's formula for unequally spaced values of argument.

(5) Construct the divided difference table for the following data:

x:	–1	2	3	6
y:	– 4	2	10	16

(6) Write the formula to find the first and second derivatives at the point $x = x_n$.

(7) Write all the subintervals of [0,18] for applying Simpson's Rule, taking $n = 6$

(8) Define : Initial Value Problem

Que:2 Attempt any TWO. **[10]**

(1) If the arguments be equally spaced, then prove that the n^{th} divided difference would be a constant.

(2) Use Lagrange's Interpolation Formula to obtain the value of $y(6)$:

x:	5	7	11	13	21
$f(x)$:	15	39.2	145.2	236.6	970.2

(3) Express the rational function $f(x) = \dfrac{x^2 + x - 3}{x^3 - 2x^2 - x + 2}$ as a sum of partial fraction.

Que:3 Attempt any TWO. **[10]**

(1) Derive the formula to find the second order differentiation at the point $x = x_0$.

(2) Find $\left[\dfrac{dy}{dx}\right]_{x=3}$ from the tabulated values:

x:	3	3.2	3.4	3.6	3.8
y:	–14	–10.032	–5.296	– 0.256	6.672

(3) The following table of values x and y is given, find the value of the second derivative when $x = 2.2$:

x :	1.4	1.6	1.8	2.0	2.2
y:	4.0552	4.9530	6.0496	7.3891	9.0250

Que:4 Attempt any TWO. **[10]**

(1) State and prove Simpson's $\frac{3}{8}$ rule.

(2) Use Trapezoidal rule to find the integral $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta$; *where* $n = 6$

(3) Use Simpson's $\frac{1}{3}$ rule to evaluate the integral $\int_1^3 \frac{1}{x^2} dx$ by taking nine intervals.

Que:5 Attempt any TWO. **[10]**

(1) Explain Picard's method to solve the initial value problem, $\frac{dy}{dx} = f(x, y)$, where $y(x_0) = y_0$.

(2) Using Taylor's series method solve the initial value problem, $\frac{dy}{dx} = y + x^2$, $y(0) = 1$. Obtain y for x = 0.01,0.02

(3) Using Euler's method solve the initial value problem, $\frac{dy}{dx} + 2y = 0$, $y(0) = 1$. Obtain $y(0.1)$, $y(0.2)$ and $y(0.3)$.

RAN-0952

RAN-0952

S.Y.B. Sc. (Sem.-IV) Examination

March / April - 2019

Mathematics Paper : MTH - 403

Introduction To Abstract Algebra

(New Course)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **S.Y.B. Sc. (Sem.-IV)**

Name of the Subject :
☛ **Mathematics Paper : MTH - 403**

Subject Code No.: 0 9 5 2

Seat No.:

Student's Signature

Instructions :

(1) All questions are compulsory.
(2) Figures to the right indicate marks of corresponding question.
(3) Follow usual notations.
(4) Use of non-programmable scientific calculator is allowed.

1. Answer the following as directed : (Any FIVE) **(10)**

(1) If $a|b$ and $a|c$, then prove that $a|b.\,x + c.\,y$, for any integers x, y.
(2) If $a.\,b = a.\,c \ (mod\ n)$ and $(a, n) = 1$, then prove that $b \equiv c \ (mod\ n)$.
(3) In a group G; prove that $(a^{-1})^{-1} = a$; for every $a \in G$,
(4) If $x = x^{-1}$; for every element x in a group G, then prove that G is abelian.
(5) Justify : The set of all prime numbers is a subgroup of the group all positive rational numbers under multiplication.
(6) Prove that a cyclic group is abelian.
(7) Define a field. Give an example of an integral domain which is not a field.
(8) In a Boolean ring R; prove that $a + a = 0$; for every a in R.

2. **Attempt any TWO :** (10)

(1) Define a prime number. If $a|b.c$ and a and b are relatively prime integers, then prove that $a|c$.

(2) If $a \equiv b \,(mod\, n)$ and $c \equiv d \,(mod\, n)$, then prove that (i) $a - c \equiv b - d \,(mod\, n)$; (ii) $a.c \equiv b.d \,(mod\, n)$.

(3) Find the integers m and n satisfying $(6540, 1206) = 6540\, m + 1206\, n$.

3. **Attempt any TWO :** (10)

(1) In a group G; prove that:

(i) Every element $a \in G$ has a unique inverse.

(ii) $(a \bullet b)^{-1} = b^{-1} \bullet a^{-1}$ *for all* $a, b \in G$.

(2) Prove that $G = \{1, 3, 5, 7\}$ is a group under the binary operation x8 ; multiplication modulo 8 .

(3) Let G be a group. Prove that G is abelian if and only if $(a.\, b)^2 = a^2 .\, b^2$; *for all* $a, b \in G$.

4. **Attempt any TWO :** (10)

(1) If H is a finite non - empty subset of a group G ; which is closed under the product in G, then prove that H is a subgroup.

(2) If H is a subgroup of a group G and $a \in G$, then prove that $aHa^{-1} = \{a.h.\, a^{-1} \in G \mid h \in H\}$ is a subgroup of G .

(3) Define the order of an element in a group. In a group G; if $a^3 = e$ and $a.\, b.\, a^{-1} = b^2$; for some $a, b \in G$, then find $o(b)$.

5. **Attempt any TWO :** (10)

(1) Define an integral domain. Prove that every field is an integral domain.

(2) Prove that the commutative ring D is an integral domain if and only if $a,b,c \in D$ with $a \neq 0$; the relation

$a.b = a.c \;\Rightarrow b = c$ holds in D.

(3) Prove that every Boolean ring is commutative.

R A N - 7 0 2 6

RAN-7026

B. Sc. Sem-IV Examination

March / April - 2019

Mathematics - MTH - 403

Numerical Analysis-II

(Old Course)

(Old or New to be mentioned where necessary)

[Total Marks: 50

સૂચના : / Instructions

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **B. Sc. Sem-IV**

Name of the Subject :

☛ **Mathematics - MTH - 403 (Numerical Analysis-II)**

Subject Code No.: 7 0 2 6

Seat No.:

Student's Signature

(1) All questions are compulsory.
(2) Follow usual notations.
(3) Figures to the right indicate total marks of the question.
(4) Use of Scientific non-programmable calculator is allowed.
(5) Total marks 50.

Que:1 **Answer any FIVE as directed.** **[10]**

(1) Use divided difference interpolation formula to obtain the function $f(x)$ for the given data (1,9) and (0,5).

(2) If $y(x) = \frac{1}{x^3}$, then find the value of [a, b].

(3) Prove that $[x_1, x_2] = [x_2, x_1]$.

(4) Write the Lagrange's formula for unequally spaced values of argument.

(5) Construct the divided difference table for the following data:

x:	0	2	3	5
y:	0	1	5	11

(6) Write the formula to find the first and second derivatives at the point $x = x_0$.

(7) Write all the subintervals of [10,16] for applying Simpson's $\frac{1}{3}$ Rule, taking $h = 0.5$

(8) Define : Initial Value Problem

Que:2 **Attempt any TWO.** **[10]**

(1) Derive Newton's Divided Difference Interpolation Formula.

(2) Use Lagrange's Interpolation formula to obtain the value of y(1):

x:	0	4	5	7
$y(x)$:	7.71	8.29	8.43	8.71

(3) Express the rational function $f(x) = \dfrac{3x^2 + x + 1}{x^3 - 6x^2 + 11x - 6}$ as a sum of partial fraction.

Que:3 **Attempt any TWO.** **[10]**

(1) Derive the formula to find the first order differentiation at the point $x = x_n$.

(2) From the following table of values, obtain $\left[\dfrac{d^2y}{dx^2}\right]_{x=0.4}$:

x:	0.1	0.2	0.3	0.4	0.5
y:	1.5	5	11.5	20	32

(3) The following table of values x and y is given, find the value of the second derivative when $x = 0.5$:

x:	1.1	1.2	1.3	1.4	1.5
y:	0.3	0.7	1.25	1.32	1.55

Que:4 **Attempt any TWO.** **[1**

(1) Derive the Simpson's $\frac{3}{8}$ rule.

(2) Use Trapezoidal rule to find the integral

$$I = \int_0^1 \frac{1}{1 + x^2}\,dx;\ \text{where } h=0.125$$

(3) Apply Simpson's $\frac{1}{3}$ rule to obtain the value of the integral

$$\int_0^2 \frac{1}{\sqrt{x}\,\sqrt{1 - x}}\,dx;\ \text{where } h = 0.25$$

Que:5 **Attempt any TWO.** **[10]**

(1) Explain Taylor's series method to solve the initial value problem $\frac{dy}{dx} = f(x, y)$, where $y(x_0) = y_0$.

(2) If $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$, compute the value of $y(2)$, $y(3)$ and $y(4)$ using Euler's method. (Where $h = 0.5$)

(3) Consider the initial value problem $\frac{dy}{dx} = y + x^2$; $y(0) = 1$, use Picard's method to obtain $y(0.5)$ and $y(1.0)$.
