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Topological Weak Specification and Distributional Chaos on Noncompact Spaces

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In this paper, we relate the topological definitions of specification property and distributional chaos defined for uniformly continuous self-maps on noncompact, nonmetrizable spaces. We prove that a uniformly continuous surjective self-map acting on a uniformly locally compact Hausdorff uniform space with topologically weak specification property and a pair of distal points is topologically distributionally chaotic of type 1. This extends the result due to Oprocha and Štefánková [2008]. As a consequence, we get that uniformly continuous surjective self-map on a uniformly locally compact totally bounded Hausdorff uniform space with topological shadowing, topological mixing, and a distal pair is topologically distributionally chaotic of type 1.

Keywords: Uniform space; topological weak specification; distributional chaos.

1. Introduction

The complexity and unpredictability of dynamical systems have been the central topic of research in the past few decades. The first mathematical definition of chaos was given by Li and Yorke [1975]. Later, Schweizer and Smítal extended Li and Yorke approach by measuring lower and upper densities of the rate of proximality of pairs, and thus introduced the much stronger notion of chaos, popularly termed as distributional chaos [Schweizer & Smítal, 1994]. Since then distributional chaos has evolved into three variants termed as $DC1$, $DC2$ and $DC3$ (ordered from strongest to weakest) [Balibrea *et al.*, 2005].

Specification property is another strong notion of chaos which was introduced by Bowen [1971]. Sklar and Smítal have proved that a continuous map defined on a compact metric space with the specification property is distributionally chaotic of type 3 [Sklar & Smítal, 2000]. Oprocha and Štefánková proved that a continuous map f acting on compact metric space with a weaker form of specification property and with a pair of distal points is distributionally chaotic in a very strong sense [Oprocha & Štefánková, 2008].

In recent years, researchers have shown great interest in studying chaos for general topological spaces, which are not necessarily compact metrizable. The metric notions of chaos were extended to uniform spaces that are not necessarily compact metrizable in [Awartani & Elaydi, 2000; Arai, 2018]. The topological notion of specification property for